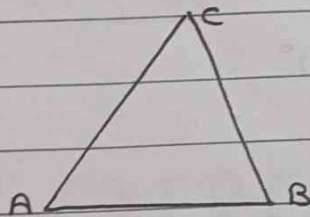


CHAPTER - 9

Triangle and its properties

Triangle - A triangle is a simple closed figure made with three line segment.



Elements of a triangle :-

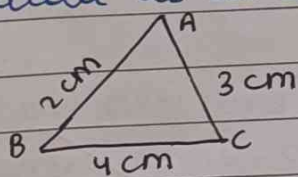
Vertices of $\triangle ABC$ - A, B, C

Sides of $\triangle ABC$ - $\overline{AB}, \overline{BC}, \overline{AC}$

Angles of $\triangle ABC$ - $\angle A, \angle B$ and $\angle C$ or $\angle CAB, \angle ABC$ and $\angle ACB$.

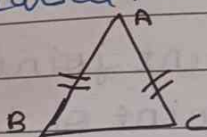
Types of triangle on basis of sides:-

- Scalene triangle - If all the sides of a triangle are unequal it is called a scalene triangle.



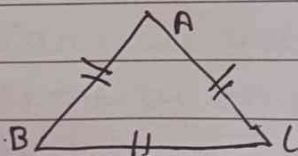
$$\overline{AB} \neq \overline{BC} \neq \overline{AC}$$

- Isosceles triangle - If any two sides of a triangle are equal, it is called an isosceles triangle.



$$AB = AC$$

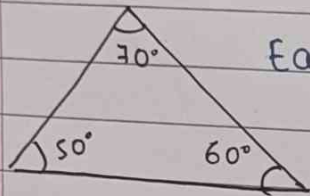
- Equilateral triangle - If all three sides of a triangle are equal it is called equilateral triangle



$$AB = BC = AC$$

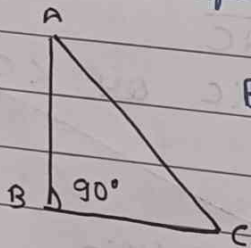
Types of triangles on the basis of angle :-

- 1) Acute angled triangle - If all three angles of a triangle are acute (less than 90°), it is called an acute angled triangle.



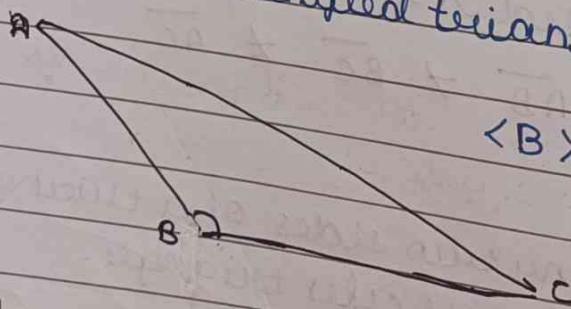
Each angle $< 90^\circ$

- 2) Right angled triangle - If one angle of the triangle is right angle ($= 90^\circ$), it is called a right angled triangle.



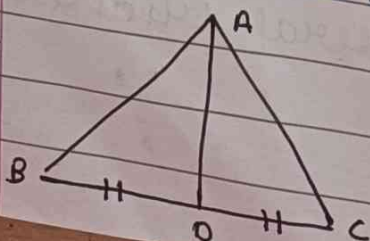
$B = 90^\circ$

- 3) Obtuse angled triangle - If one angle of a triangle is greater than 90° , it is called an obtuse angled triangle.



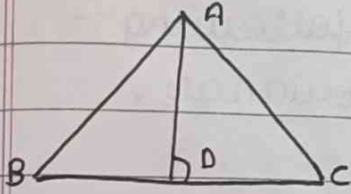
$\angle B > 90^\circ$

Median of a triangle - The line segment joining the vertex of a triangle to the mid-point of the opposite side is called median of the triangle.



AD is median
 $BD = DC$

Altitudes of a triangle - The perpendicular line segment from a vertex of a triangle to the opposite side is called an altitude of triangle.

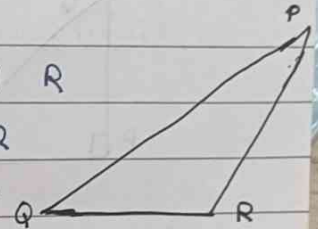


Altitude = height
Perpendicular
AD is $\perp$ to BC

EXERCISE 9.1

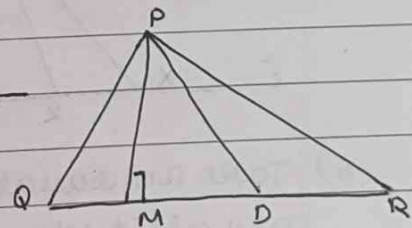
1) In the adjoining figure:

- Name the vertex opposite to side PQ R
- Name the side opposite to vertex Q PR
- Name the angle opposite to side QR $\angle P$
- Name the side opposite to $\angle R$ PQ



2) In the adjoining $\triangle PQR$, if D is the mid point of QR and $PM \perp QR$ then.

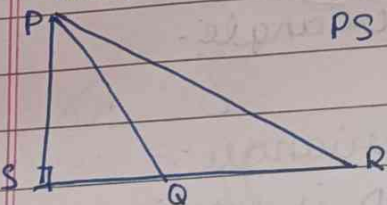
- $\overline{PM}$ is Altitude
 - $\overline{PD}$ is Median
- Is $QM \neq MR$?



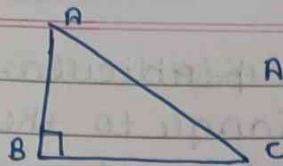
3) Will an altitude always lie in the interior of a triangle? If no, draw a rough sketch to show such a case.

No,

PS is the altitude.



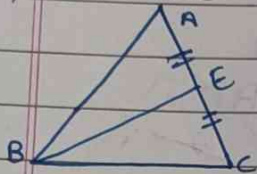
4) Can you think of a triangle in which two altitudes of the triangle are its sides? If yes draw a rough sketch to show such a case.



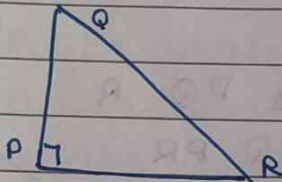
AB and BC are altitudes of $\triangle ABC$
Yes, it is right angled triangle.

5) Draw rough sketches for the following :-

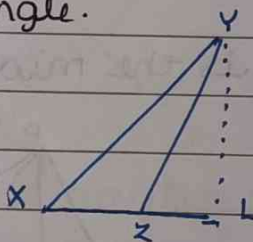
i) In $\triangle ABC$, BE is a median of the triangle.



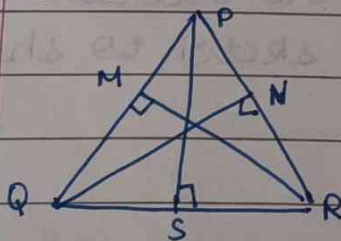
ii) In $\triangle PQR$, PQ and PR are altitudes of the triangle.



iii) In $\triangle XYZ$, YL is an altitude in the exterior of the triangle.

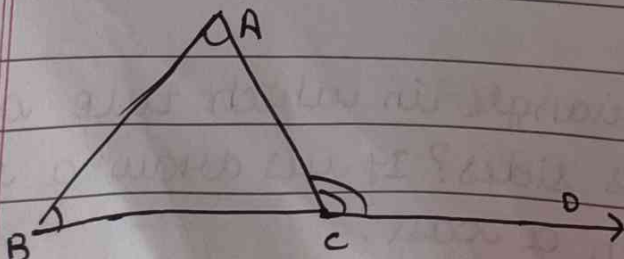


6) Take an equilateral triangle and draw its median and altitude and check that the medians and altitudes are same.



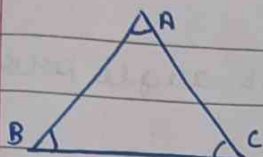
The altitude of an equilateral triangle divide the sides into equal parts.
Hence, altitudes are also the medians of the triangle.

Exterior angle property of a triangle :-



$\angle ACD$ is exterior angle
 $\angle ACD = \angle A + \angle B$

Angle sum property of a triangle:-



$$\angle A + \angle B + \angle C = 180^\circ$$

EXERCISE 9.2

1) Find the value of the unknown exterior angle x in each of the following figures:

i)

$$x = 45 + 65 \text{ (Exterior angle property)}$$

$$x = 110$$

ii)

$$x = 55 + 40 \text{ (Exterior angle property)}$$

$$x = 95^\circ$$

iii)

$$x = 50 + 50 \text{ (Exterior angle)}$$

$$x = 100^\circ$$

2) Find the value of the unknown interior angle x in each of the following figures:

i)

$$x + 50 = 115^\circ$$

$$x = 115 - 50 = 65^\circ$$

ii)

$$x + 30 = 80$$

$$x = 80 - 30$$

$$x = 50^\circ$$

iii)

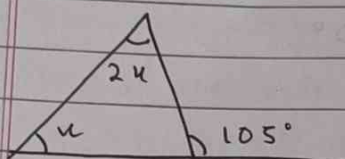
$$x + 36 = 70$$

$$x = 70 - 36$$

$$x = 34^\circ$$

3) Find the value of x in each of the following figures:

i)

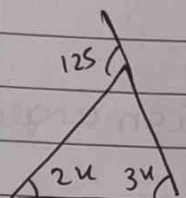


$$x + 2x = 105 \text{ (Ext angle prop)}$$

$$3x = 105$$

$$x = \frac{105}{3} = \boxed{x = 35}$$

ii)

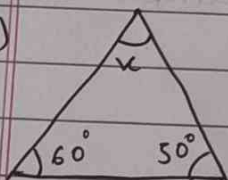


$$2x + 3x = 125 \text{ (Ext angle prop)}$$

$$5x = 125 \Rightarrow x = \frac{125}{5}$$

$$\boxed{x = 25^\circ}$$

iii)

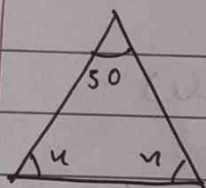


$$x + 50 + 60 = 180^\circ \text{ (Angle sum property)}$$

$$x + 110 = 180$$

$$x = 180 - 110 \Rightarrow \boxed{x = 70^\circ}$$

4) Find the value of unknown x in each of the following figures:

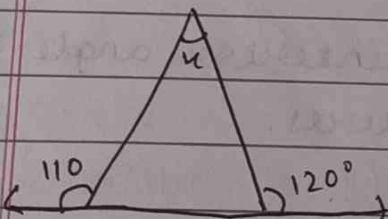


$$x + x + 50 = 180^\circ \text{ (Angle sum prop)}$$

$$2x = 180 - 50$$

$$x = \frac{130}{2} = 65^\circ \Rightarrow \boxed{x = 65^\circ}$$

i)



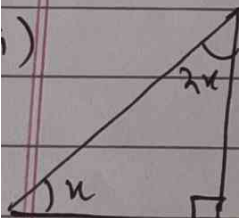
$$x + 110 + 120 = 180$$

$$x = 180 - 230 \Rightarrow x = -50$$

$$x + 70 = 120$$

$$x = 120 - 70 \Rightarrow \boxed{x = 50^\circ}$$

ii)



$$90 + 2x + x = 180$$

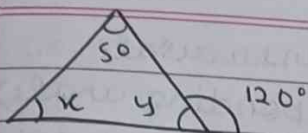
$$90 + 3x = 180$$

$$3x = 180 - 90 \Rightarrow x = \frac{90}{3} = 30^\circ$$

$$\boxed{x = 30^\circ}$$

Find the value of x and y in each of the following figures.

i)



$$y + 120 = 180 \text{ (L.P.)}$$

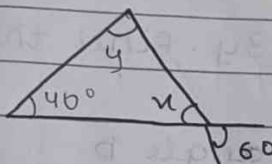
$$y = 180 - 120 \Rightarrow \boxed{y = 60}$$

$$x + y + 50 = 180^\circ \text{ (ASP)}$$

$$x + 60 + 50 = 180$$

$$x = 180 - 110 \Rightarrow \boxed{x = 70^\circ}$$

ii)



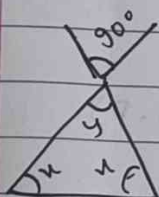
$$x = 60^\circ \text{ (V.O.A)}$$

$$60^\circ + 40^\circ + y = 180$$

$$100 + y = 180$$

$$y = 180 - 100 \Rightarrow \boxed{y = 80}$$

iii)



$$y = 90^\circ \text{ (V.O.A)}$$

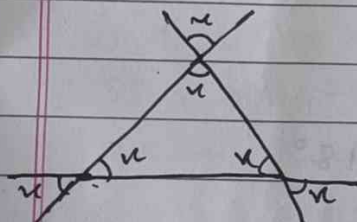
$$x + x + y = 180 \text{ (ASP)}$$

$$2x + 90 = 180$$

$$2x = 180 - 90$$

$$\boxed{x = 45^\circ}$$

6) Find the value of x and y in each of the following figures.



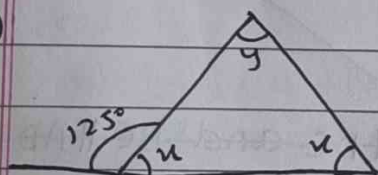
$$y = x$$

$$x + x + y = 180^\circ \text{ (Angle sum property)}$$

$$3x = 180 \Rightarrow x = \frac{180}{3} = 60^\circ$$

$$\boxed{x = 60^\circ}$$

ii)

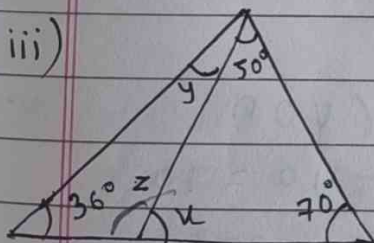


$$x = 180 - 125 = 55^\circ \text{ (L.P.)}$$

$$55 + 55 + y = 180$$

$$y = 180 - 110 = 70^\circ$$

iii)



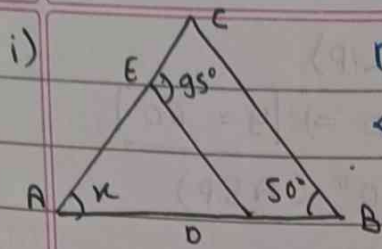
$$\angle z = 70 + 50 = 120^\circ \text{ (Ext ang prop)}$$

$$x = 180 - 120 = 60^\circ$$

$$y + 36 + 120 = 180$$

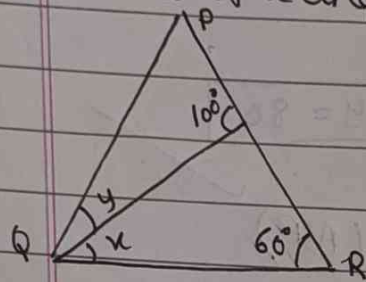
$$y = 180 - 156 = 24^\circ$$

7) a) In the fig (i) given below, $DE \parallel BC$. Find the value of x .



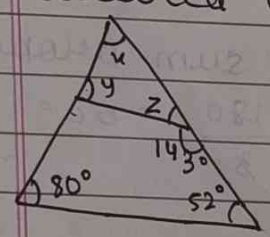
i) $DE \parallel BC$, AB is transversal.
 $\angle EDA = 50^\circ$ (Corresponding angles)
 $\angle CED = x + \angle EDA$ (Ext angle prop)
 $95^\circ = x + 50^\circ$
 $x = 45^\circ$

b) In the fig (ii) given below, $\angle P = 3y$. Find the values of x and y .



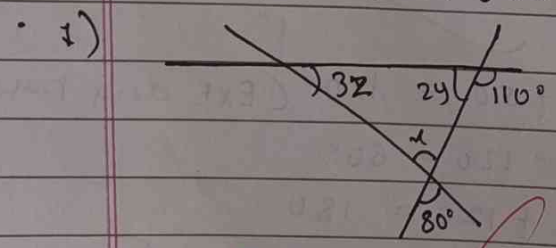
$100^\circ = x + 60^\circ$ (Ext angle prop)
 $x = 40^\circ$
 In ΔPQR
 $y + \angle P + 100 = 180^\circ$ (A.S.P)
 $y + 3y + 100 = 180$
 $4y = 180 - 100$
 $y = \frac{80}{4} = 20$ | $y = 20^\circ$

c) In the fig (iii) given below, find the size of each lettered angle.



$\angle z = 180 - 143 = 37^\circ$
 $\angle x = 180 - (80 + 52) = 48^\circ$
 $y + 37 + 48 = 180$
 $\angle y = 180 - 85 = 95^\circ$

8) In the fig 1) given below, $AB \parallel DC$ and $BC \parallel AD$. Find the value of x, y, z .



$x = 80^\circ$ (V.O.A)
 $2y = 180 - 110 = 70$
 $y = \frac{70}{2} = 35^\circ$
 $3z + 2y + x = 180$
 $3z + 70^\circ + 80^\circ = 180^\circ$
 $3z = 180 - 150 = 30$
 $z = \frac{30}{3} = 10^\circ$

b) In the fig (2) given below find:-

i) $\angle PTQ$

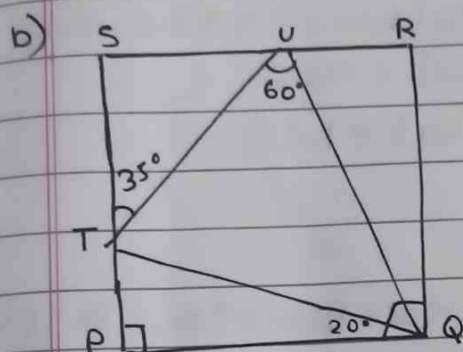
ii) $\angle QTV$

iii) $\angle SUT$

iv) $\angle RUQ$

v) $\angle RQU$

vi) $\angle UQT$



i) $\angle PTQ$

$$90^\circ + 20^\circ + \angle PTQ = 180^\circ \text{ (A.S.P)}$$

$$110^\circ + \angle PTQ = 180^\circ$$

$$\boxed{\angle PTQ = 70^\circ}$$

ii) $\angle QTV$

$$\angle PTQ + \angle QTV + \angle STU = 180^\circ \text{ (L.P)}$$

$$70^\circ + \angle QTV + 35^\circ = 180^\circ$$

$$\angle QTV = 180 - 105 \quad \boxed{\angle QTV = 75^\circ}$$

iii) $\angle SUT$

$$90^\circ + 35^\circ + \angle SUT = 180^\circ \text{ (A.S.P)}$$

$$125^\circ + \angle SUT = 180^\circ$$

$$\angle SUT = 180 - 125 \quad \boxed{\angle SUT = 55^\circ}$$

iv) $\angle SUT + \angle TUQ + \angle RUQ = 180^\circ \text{ (Linear pair)}$

$$55^\circ + 60^\circ + \angle RUQ = 180^\circ$$

$$115^\circ + \angle RUQ = 180^\circ$$

$$\angle RUQ = 180 - 115 \quad \boxed{\angle RUQ = 65^\circ}$$

v) $\angle RQU$

$$\angle R + \angle QUR + \angle RQU = 180^\circ \text{ (A.S.P)}$$

$$90^\circ + 65^\circ + \angle RQU = 180^\circ$$

$$\angle RQU = 180 - 155 \quad \boxed{\angle RQU = 25^\circ}$$

vi) $\angle UQT$

$$\angle PQT + \angle TQU + \angle UQR = 90^\circ$$

$$20^\circ + \angle UQT + 25^\circ = 90^\circ$$

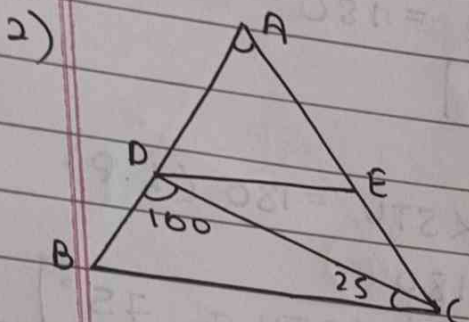
$$\angle UQT = 90 - 45 \quad \boxed{\angle UQT = 45^\circ}$$

9) a) In the fig 1) given below, $AB \parallel DC$ and $BC \parallel AD$. Find

- i) $\angle DBC$ ii) $\angle BDC$ iii) $\angle DBA$ iv) $\angle A$

b) In the given fig (2) given below, $DE \parallel BC$. Find

- i) $\angle B$ ii) $\angle EDC$ iii) $\angle ADE$
 iv) $\angle AED$ v) $\angle DEC$ vi) $\angle DCE$



i) $\angle B$
 $\angle B + \angle BDC + \angle DCB = 180^\circ$ (A.S.P)
 $\angle B + 100^\circ + 25^\circ = 180^\circ$

$\angle B = 180 - 125$ $\angle B = 55^\circ$

ii) $\angle EDC$ $\Rightarrow DE \parallel BC$

$\angle EDC = 25$ (Alternate angle)

iii) $\angle ADE$

$\angle ADE + 25 + 100 = 180$ (L.P)
 $\angle ADE = 180 - 125 = 55^\circ$

iv) $\angle AED$ $\Rightarrow$ In $\triangle ADE$

$\angle A + \angle ADE + \angle AED = 180^\circ$ (A.S.P)
 $55^\circ + 55^\circ + \angle AED = 180$

$\angle AED = 180 - 110$ $\angle AED = 70^\circ$

v) $\angle DEC$

$\angle AED + \angle DEC = 180$ (L.P)

$70 + \angle DEC = 180$

$\angle DEC = 180 - 70$ $\angle DEC = 110^\circ$

vi) $\angle DCE$

$\angle DCE + \angle DEC + \angle CDE = 180^\circ$ (A.S.P)

$110^\circ + \angle DCE + 25^\circ = 180^\circ$

$\angle DCE = 180 - 135$ $\angle DCE = 45^\circ$

- 10) One of the angles of a triangle measures 80° and the other two angles are equal. Find the measure of each of the equal angles.

Sol.

Let the angle be x

$$80 + x + x = 180$$

$$2x = 180 - 80$$

$$x = \frac{100}{2} = \boxed{x = 50^\circ}$$

- 11) If one angle of a triangle is 60° and the other two angles are in the ratio $2:3$, find the angles.

Sol. Let the angle be x

Second angle be $2x$

Third angle be $3x$

$$x + 2x + 3x = 180^\circ$$

$$5x = 120^\circ$$

$$\boxed{x = 24^\circ}$$

$$2x = 2 \times 24 = 48^\circ$$

$$3x = 3 \times 24 = 72^\circ$$

- 12) If the angles of a triangle are in the ratio $1:2:3$, find the angles. Classify the angle in two different ways:-

Sol. Let the angle be x

Second angle be $2x$

Third angle be $3x$

$$x + 2x + 3x = 180^\circ$$

$$6x = 180$$

$$\boxed{x = 30^\circ}$$

$$2x = 2 \times 30 = 60^\circ$$

$$3x = 3 \times 30 = 90^\circ$$

Scalene and right angle triangle.

13) Can a triangle have three angles whose measures are?

i) $65^\circ, 74^\circ, 39^\circ$?

$$65 + 74 + 39 \neq 180^\circ$$

$$178^\circ \neq 180^\circ$$

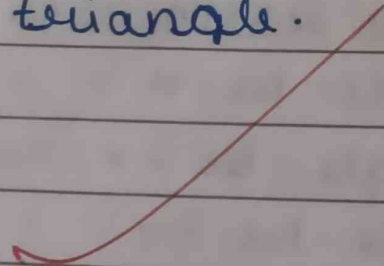
No!.

ii) $\frac{1}{3} \times 90 = 30^\circ$

$$30, 90, 60 \Rightarrow 30 + 90 + 60 = 180^\circ$$

The sum of angles of triangles is 180° .

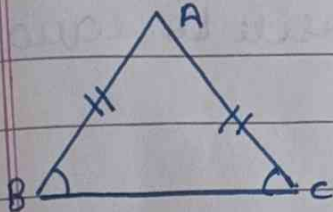
Yes, it forms a triangle.



CONCEPT

ANGLE PROPERTY OF SPECIAL TRIANGLE :-

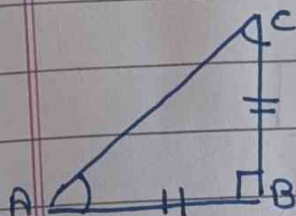
- Isosceles triangle :-



If $AB = AC$ } The angles opposite to equal sides are equal
 $\angle C = \angle B$

If $\angle B = \angle C$ } The sides opposite to equal angles are equal.
 $AC = AB$

- Isosceles right angle triangle :-



$$\angle B = 90^\circ \Rightarrow AB = BC$$

$\angle C = \angle A$ (Angles opposite to equal sides are equal)

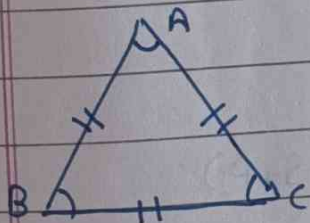
$$\angle A + \angle B + \angle C = 180^\circ \text{ (A.S.P.)}$$

$$\angle A + 90^\circ + \angle A = 180^\circ$$

$$2\angle A = 90^\circ \Rightarrow \angle A = \frac{90}{2} = 45^\circ$$

$$\begin{aligned} \angle A &= \angle C \\ \angle C &= 45^\circ \end{aligned} \Rightarrow \angle A = 45^\circ, \angle B = 90^\circ, \angle C = 45^\circ$$

- Equilateral triangle :-



$$AB = BC = CA$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle A + \angle A = 180^\circ$$

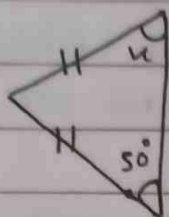
$$3\angle A = 180^\circ \mid \angle A = 60^\circ$$

$$\angle A = \angle B = \angle C = 60^\circ$$

EXERCISE 9.3

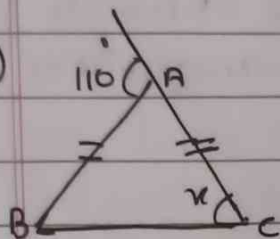
1) Find the value of x in each of the following:

i)



$x = 50$ (Angles opposite to equal side are equal)

ii)



$$AB = AC$$

$$\angle C = \angle B = x$$

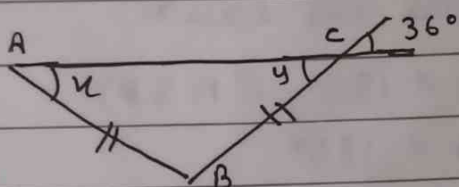
$$110^\circ = \angle B + \angle C$$

$$110^\circ = x + x$$

$$2x = 110^\circ$$

$$x = 55^\circ$$

iii)



$$y = 36^\circ \text{ (VOA)}$$

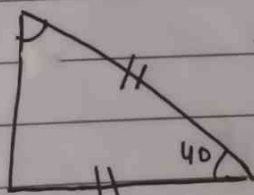
$$AB = BC$$

$$x = 36^\circ \text{ (If sides are equal then angles are also equal)}$$

then angles are also equal

2) Find the value of x in each of the following figures:

i)



$$AC = BC$$

$$\angle B = \angle A = x$$

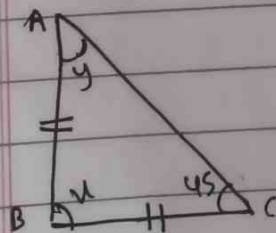
$$\text{In } \triangle ABC$$

$$x + x + 40^\circ = 180^\circ \text{ (A.S.P)}$$

$$2x = 180 - 40$$

$$x = \frac{140}{2} \Rightarrow x = 70^\circ$$

ii)

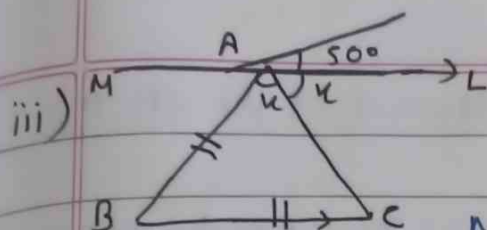


$$AB = BC$$

$$y = 45^\circ$$

$$x = 180 - 45 + 45$$

$$= 90^\circ$$



$$AB = BC$$

$$\angle C = \angle A = x$$

$ML \parallel BC$, AC is transversal

$\angle LAC = \angle BCA$ (Alternate angle)

$$\angle LAC = x^\circ$$

$$x + x + 50^\circ = 180^\circ \text{ (Linear pair)}$$

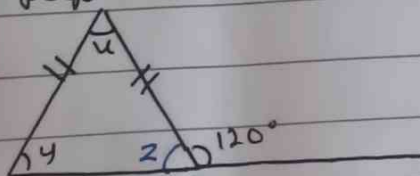
$$2x + 50 = 180^\circ$$

$$2x = 180 - 50^\circ$$

$$2x = 130 \Rightarrow \boxed{x = 65^\circ}$$

3) Find the values of x and y in each of the following figures:

i)



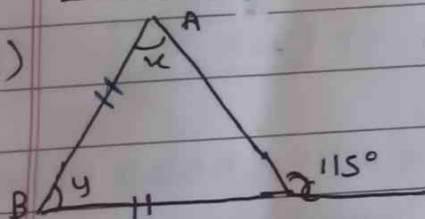
$$\angle z = 180 - 120 = 60^\circ \quad \boxed{z = 60^\circ}$$

$$\angle y = 60^\circ \text{ (as two sides are equal)}$$

$$x = 180 - 60 - 60$$

$$\boxed{x = 60^\circ}$$

ii)



$AB = BC$ (angle opposite are equal)

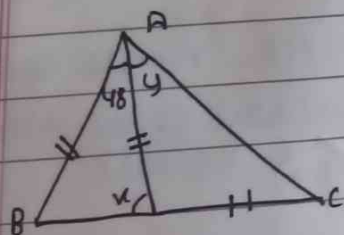
$$x + 115 = 180 \text{ (L.P.)}$$

$$x = 180 - 115 \Rightarrow \boxed{x = 65^\circ}$$

In $\triangle ABC$

$$\angle y = 115 - 65 = \boxed{y = 40^\circ}$$

iii)



In $\triangle ABC \Rightarrow AB = AC$ (Angles opposite to equal sides are equal)

$$\angle ABC = \angle ACB = x$$

In $\triangle ABC$

$$48 + x + x = 180 \text{ (A.S.P.)}$$

$$2x = 180 - 48 \Rightarrow 2x = 132$$

$$x = \frac{132}{2} = \boxed{x = 66^\circ}$$

In $\triangle ACD$

$$y + y + 114 = 180$$

$$2y + 114 = 180$$

$$2y = 180 - 114$$

$$\boxed{y = 33^\circ}$$

In $\triangle ACD \Rightarrow AC = CD$ (Angle opposite to equal sides are equal)

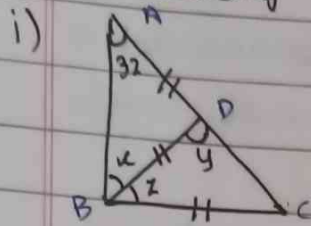
$$\angle D = \angle CAD = y$$

$$\angle ACB + \angle ACD = 180 \text{ (L.P.)}$$

$$x = \angle ACD = 180^\circ$$

$$66 + \angle ACD = 180^\circ \Rightarrow \angle ACD = 180 - 66 = 114^\circ$$

4) Calculate the size of each lettered angle in the following figures:



In $\triangle ABD \Rightarrow AD = BD$ (Angle opposite to equal sides are equal)
 $x = 32^\circ$

In $\triangle ABD$
 $32 + x + \angle BDA = 180$ (A.S.P)

$$32^\circ + 32^\circ + \angle BDA = 180$$

$$\angle BDA = 180 - 64 \Rightarrow \angle BDA = 116^\circ$$

$$y + 116 = 180 \text{ (L.P)}$$

$$y = 180 - 116 \Rightarrow y = 64^\circ$$

In $\triangle BDC$

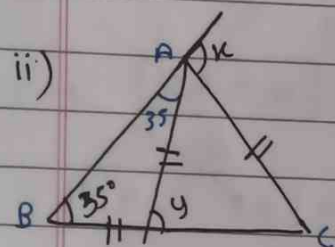
$BD = BC$ (Angles opposite to equal sides are equal)

$$\angle BCD = y = 64^\circ$$

In $\triangle BCD$

$$64 + 64 + z = 180 \text{ (A.S.P)}$$

$$128 + z = 180 \Rightarrow z = 180 - 128 \Rightarrow z = 52$$



In $\triangle ABC \Rightarrow BC = AC$ (Angle opposite to equal sides are equal)
 $\angle BAC = 35^\circ$

$$y = 35 + 55^\circ \text{ (Ext angle property)}$$

In $\triangle ACD \Rightarrow AC = AD$ (Angle opposite to equal sides are equal)
 $y = \angle ADC = 70^\circ$

$$x = 35 + 70^\circ \text{ (Ext angle property)}$$

$$x = 105^\circ$$

5) If the angles of a triangle are in the ratio 1:2:3, find all the angles of the triangle. Classify the triangle in two different ways.

Let the angle be x

$$x + 2x + x = 180 \text{ (A.S.P)}$$

$$4x = 180$$

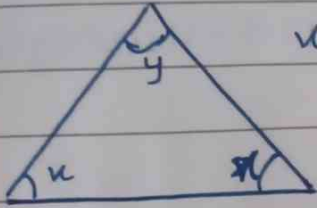
$$x = 45^\circ$$

$$2x = 2 \times 45 = 90^\circ$$

$$45^\circ, 90^\circ, 45^\circ$$

Isosceles and right angle triangle

- 6) In an isosceles triangle, a base angle is four times its vertical angle. Find all the angles of the triangle.



x is a base angle. y is vertical angle

$$x = 4y$$

$$x + x + y = 180^\circ \text{ (A.S.P)}$$

$$4y + 4y + y = 180^\circ$$

$$9y = 180$$

$$y = \frac{180}{9} = 20^\circ \quad [y = 20^\circ]$$

$$x = 4y$$

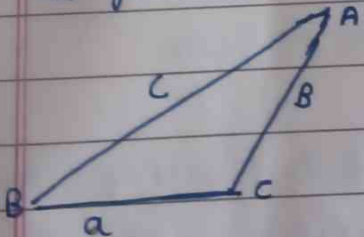
$$x = 4 \times 20 = 80 \Rightarrow [x = 80^\circ]$$

Angles will be $80^\circ, 20^\circ, 80^\circ$

CONCEPT

SUM OF LENGTH OF TWO SIDES OF A TRIANGLE :-

The sum of length of any two sides of a triangle is greater than the length of the third side.



$$a + b > c$$

$$b + c > a$$

$$a + c > b$$

EXERCISE 9.4

- 1) Is it possible to have a triangle with the following sides:

- i) 2 cm, 3 cm, 5 cm

$$2 \text{ cm} + 3 \text{ cm} > 5 \text{ cm}$$

$$5 \text{ cm} > 5 \text{ cm}$$

No, it is not possible to form a triangle.

ii) 2.5 cm, 4.5 cm, 8 cm

$$2.5 + 4.5 > 8$$

$7 > 8$ Not possible to form a triangle.

iii) 10.2 cm, 5.8 cm, 4.5 cm

$$10.2 \text{ cm} + 5.8 \text{ cm} > 4.5 \text{ cm}$$

$$16 \text{ cm} > 4.5 \text{ cm}$$

It is possible to form a triangle.

iv) 3.4 cm, 4.7 cm, 6.2 cm

$$3.4 + 4.7 > 6.2$$

$$8.1 \text{ cm} > 6.2 \text{ cm}$$

Yes, it forms a triangle.

2) If the length of two sides of a triangle are 7 cm and 10 cm, then what can be the length of the third side?

Given sides are 7 cm and 10 cm

$$\text{Difference of sides} = 10 \text{ cm} - 7 \text{ cm} = 3 \text{ cm}$$

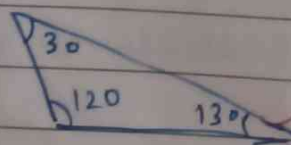
$$\text{Sum of sides} = 10 + 7 = 17 \text{ cm}$$

then we, length of third side of a triangle must be greater than 3 cm and less than 17 cm.

3) We know that in a triangle, the sum of the lengths of any two sides is greater than the length of the third side. Is the sum of any angles of a triangle also greater than the third angle? If no, draw a rough sketch to show such a case.

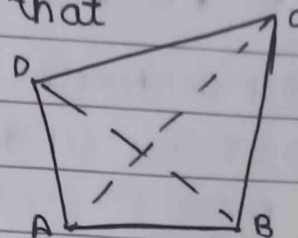
No,

$$30 + 30 = 60^\circ < 120^\circ$$



4) In the given figure, AC and BD are diagonals of a quadrilateral ABCD. Show that

$$AB + BC + CD + DA > AC + BD$$



In $\triangle ABC$

$$AB + BC > AC \quad \text{--- eq ①}$$

In $\triangle BCD$

$$BC + CD > BD$$

In $\triangle ADC$

$$AD + CD > AC \quad \text{--- Eq ②}$$

In $\triangle BAD$

$$AB + AD > BD \quad \text{--- eq ④}$$

we know sum of two sides of a triangle is greater than the third side.

$$\text{--- Eq ③}$$

Now adding eqⁿ ①, ②, ③, ④

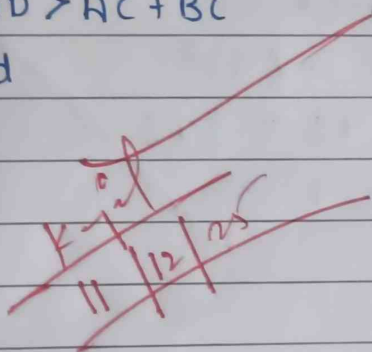
Adding eq ① ② ③ ④

$$AB + BC + BC + CD + AD + CD + AB + AD > AC + BD + AC + BD$$

$$2(AB + BC + CD + AD) > 2(AC + BD)$$

$$AB + BC + CD + AD > AC + BD$$

Hence proved



EXERCISE 9.5

1. PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

$$QR^2 = PQ^2 + PR^2 \quad (\text{Pythagoras property})$$

$$(10)^2 + (24)^2$$

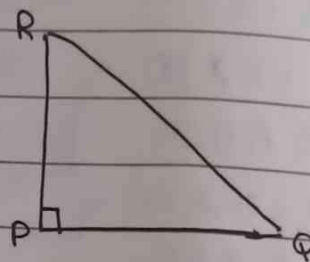
$$= 100 + 576$$

$$= 676$$

$$= 26 \times 26$$

$$QR^2 = (26)^2$$

$$QR = 26 \text{ cm}$$



- 2) ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC

$$AB^2 = BC^2 + AC^2$$

$$BC^2 = AB^2 - AC^2$$

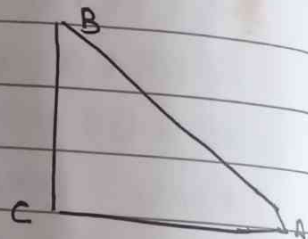
$$BC^2 = (25)^2 - (7)^2$$

$$625 - 49 = 576$$

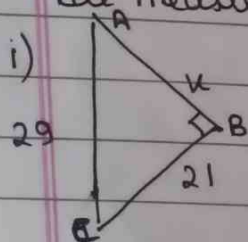
$$BC^2 = 24 \times 24$$

$$BC^2 = (24)^2$$

$$BC = 24 \text{ cm}$$



- 3) Find the value of x in each of the following figures. All measurements are in centimeters.



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 - (BC)^2 = (AB)^2$$

$$(AB)^2 = (AC)^2 - (BC)^2$$

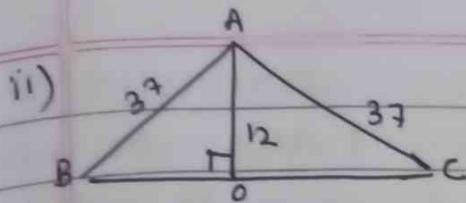
$$(x)^2 = (29)^2 - (21)^2$$

$$(x)^2 = 841 - 441$$

$$(x)^2 = 400$$

$$(x)^2 = (20)^2$$

$$x = 20$$



$$(AB)^2 = (AO)^2 + (BO)^2$$

$$(BO)^2 = (AC)^2 - (AO)^2$$

$$= (37)^2 - (12)^2$$

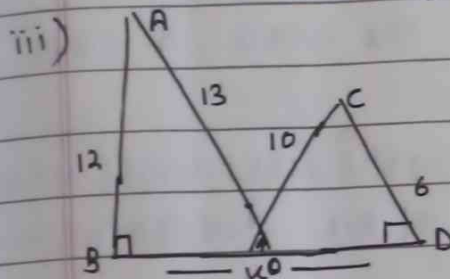
$$1369 - 144 = 1225$$

$$(BO)^2 = (35)^2$$

$$\boxed{BO = 35}$$

$$BO = OC \Rightarrow 35$$

$$BC = 35 + 35 = 70 \quad | \quad x = 70 |$$



$$(AO)^2 = (AB)^2 + (BO)^2$$

$$(BO)^2 = (AC)^2 - (AB)^2$$

$$= (13)^2 - (12)^2$$

$$= 169 - 144 = 25$$

$$(BO)^2 = (5)^2$$

$$\boxed{BO = 5}$$

$$BO = OD = 5 \quad (OC)^2 = (OD)^2 + (DC)^2$$

$$x = 5 + 5 \quad (10)^2 = (OD)^2 + (DC)^2$$

$$(10)^2 - (6)^2$$

$$100 - 36 = 64$$

$$(OD)^2 = (8)^2 \Rightarrow \boxed{OD = 8}$$

$$x = BO + OD \Rightarrow 5 + 8 = 13$$

4) Which of the following can be the sides of a right angled triangle?

1) 4 cm, 5 cm, 7 cm

(Longest side)² = Sum of the squares of other two sides

$$(4)^2 + (5)^2 = (7)^2$$

$$16 + 25 = 49$$

41 $\neq$ 49 (Pythagoras property not satisfied, so

given sides do not form a right angled triangle.

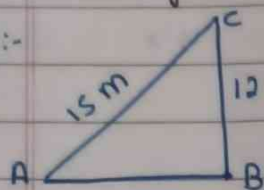
2) 1.5 cm, 2 cm, 2.5 cm.

$$(2.5)^2 \neq (1.5)^2 + (2)^2$$

$$6.25 = 2.25 + 4$$

6.25 = 6.25 A right angled triangle can be formed

- 5) A 15 m long ladder reaches a window 12 m high from the ground on placing it against a wall. How far is the foot of the ladder from the wall?



In right angled triangle ABC

$$(AC)^2 = (AB)^2 + (BC)^2$$

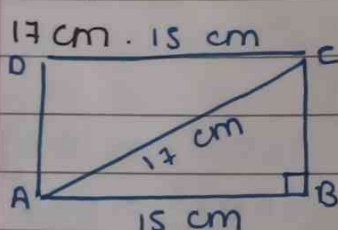
$$(15)^2 = (AB)^2 + (12)^2$$

$$225 = (AB)^2 + 144$$

$$(AB)^2 = 81 \quad \boxed{AB = 9 \text{ m}}$$

The foot of the ladder from the wall is 9 m

- 6) Find the area and perimeter of the rectangle whose length is 15 cm and the length of one diagonal is 17 cm.



$$\angle B = 90^\circ$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(17)^2 = (15)^2 + (BC)^2$$

$$289 = 225 + (BC)^2$$

$$(BC)^2 = 289 - 225 \quad \boxed{BC = \frac{64}{2}}$$

$$BC = 8 \text{ cm.}$$

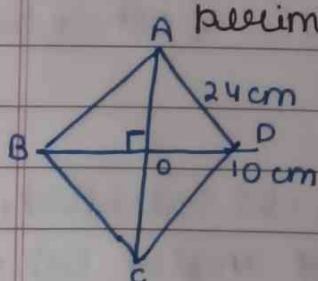
$$\text{Perimeter of rectangle} = 2 \times (L + B)$$

$$= 2 \times (15 + 8) = 46 \text{ cm}$$

$$\text{Area of rectangle} = L \times B = 15 \times 8$$

$$= 120 \text{ cm}^2$$

- 7) If the diagonal measure 10 cm and 24 cm, find its perimeter.



ABCD is rhombus

$$AO = 24 \text{ cm}$$

$$BO = 10 \text{ cm}$$

$$OA = \frac{AC}{2} = \frac{24}{2} = 12 \text{ cm}$$

$$OB = \frac{BD}{2} = \frac{10}{2} = 5 \text{ cm}$$

In $\triangle AOB$

$$(AB)^2 = (OA)^2 + (OB)^2$$

$$(AB)^2 = (12)^2 + (5)^2$$

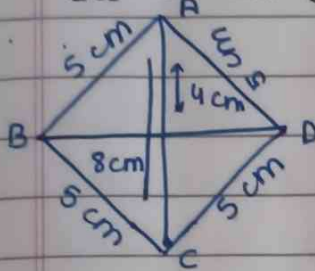
$$(AB)^2 = 144 + 25 \Rightarrow (AB)^2 = 169$$

$$(AB) = 13 \text{ cm}$$

$$\text{Perimeter} = 13 + 13 + 13 + 13$$

$$= 52 \text{ cm.}$$

- 8) The side of a rhombus is 5 cm. If the length of one diagonal of the rhombus is 8 cm, then find the length of the other diagonal.



$$AB = BC = CD = AD = 5 \text{ cm}$$

$$AC = 8 \text{ cm}$$

$$OA = \frac{AC}{2} = \frac{8}{2} = 4 \text{ cm}$$

In $\triangle OAB$

$$(AB)^2 = (OB)^2 + (OA)^2$$

$$(5)^2 = (OB)^2 + (4)^2$$

$$(OB)^2 = 25 - 16 \Rightarrow (OB)^2 = 9 \text{ cm}$$

$$OB = 3 \text{ cm.}$$

$$BD = 2 \times OB \Rightarrow 2 \times 3$$

$$BD = 6 \text{ cm}$$

- Q) iii) ~~1.5 cm, 2 cm, 2.5~~

$$7 \text{ cm, } 5.6, 4.2 \text{ cm}$$

$$(7)^2 = (5.6)^2 + (4.2)^2$$

$$49 = 31.36 + 17.64$$

$$49 = 49$$

A right angled triangle can be formed.

