SH. S. N. SIDHESHWAR SR. SEC PUBLIC SCHOOL, SEC 9-A

Holidays Homework

Class XII (Mathematics)

Holiday's Home Work:

- *Prepare an project file containing atleast 10 activities, as discussed in the class.*
- *Complete the given assignment neatly in a separate register/file.*

CLASS XII MATHEMATICS

HOLIDAYS HOMEWORK

INVERSE TRIGONOMETRY

SECTION – A (1 MARK EACH)

Q1. The value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is:

(a) $\frac{4\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) π

Q2. The value of $\cos\left(\tan^{-1}\frac{3}{4}\right)$ is:

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$

Q3. Given below are two statements: one is labelled as **Assertion A** and other is labelled as **Reason R.**

Assertion (A) : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

a. Both **A** and **R** are correct and **R** is the correct explanation of **A**
b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c. **A** is correct but **R** is not correct
d. **A** is not correct but **R** is correct

SECTION – B (2 MARKS EACH)

Q4.	If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ and $b = \tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ then find the value of a + b.
Q5.	Find the domain of $y = \sin^{-1}(x^2 - 4)$.

SECTION – C (3 MARKS EACH)

Q6.	Find the value of $\left[\sin^2\left\{\cos^{-1}\left(\frac{3}{5}\right)\right\} + \tan^2\left\{\sec^{-1}(3)\right\}\right]$.
Q7.	Simplify : $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]; \frac{1}{2} \le x \le 1$
Q8.	Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

SECTION – D (5 MARKS EACH)

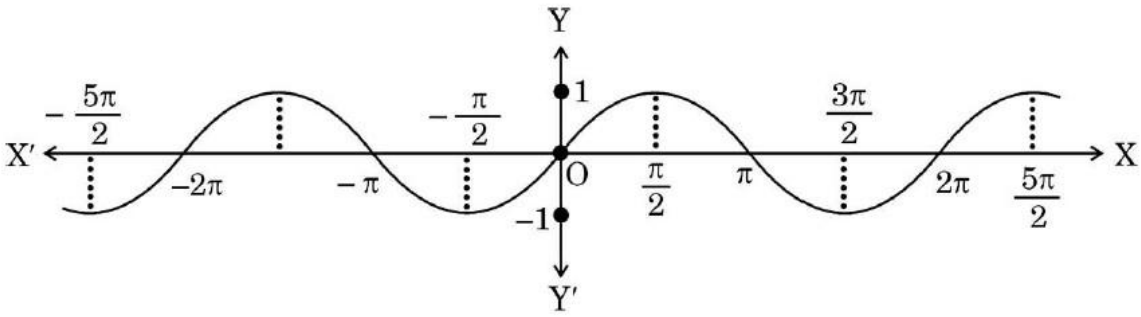
Q9.	**(a)** Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. **(b)** Show that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, \; x \in \left(0, \frac{\pi}{4}\right)$
Q10.	**(a)** Write $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right], x \neq 0$ in the simplest form. **(b)** Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

SECTION – E (4 MARKS – CASE STUDY)

Q11.

If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function.



$y = \sin x$

Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. *2*

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

ANSWERS

INVERSE TRIGONOMETRY

Q. NO.	ANSWER
1	d
2	b
3	c
4	$\frac{7\pi}{12}$
5	Domain is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$
6	$\frac{216}{25}$
7	$\frac{\pi}{3}$
8	Range $= \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
9	(to prove)
10	$\frac{1}{2}\tan^{-1}x$
11	**(i)** $A = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ or, $A = \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ or, $A = \left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$ etc. $(ii) -\frac{2\pi}{3}$ **(iii)** domain of given function is $x \in [0, 2]$ **and** range of the function is $[-\pi, \pi]$

MATRICES

SECTION – A (1 MARK EACH)

Q1. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q2. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$, then value of x for which $A^2 = B$ is :

(A) -2 (B) 2

(C) 2 or -2 (D) 4

Q3. If a matrix has 36 elements, the number of possible orders it can have, is :

(A) 13 (B) 3

(C) 5 (D) 9

Q4. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is ***false*** ?

(A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$ (D) $a_{31} = 0$

Q5. The number of all scalar matrices of order 3, with each entry – 1, 0 or 1, is :

(A) 1 (B) 3

(C) 2 (D) 3^9

Q6. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q7. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is

(a) 7 (b) 6 (c) 8 (d) 18

Q8. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\ i, j$

(C) $a_{ij} = 0, \forall\ i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

Given below are two statements: one is labelled as **Assertion A** and other is labelled as **Reason R.**

In the light of the given statements, choose the ***most appropriate*** answer from the options given below

a. Both **A** and **R** are correct and **R** is the correct explanation of **A**
b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c. **A** is correct but **R** is not correct
d. **A** is not correct but **R** is correct

Q9. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

Q10. *Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

SECTION – B (2 MARKS EACH)

Q11. Matrix $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$ is given to be symmetric, find values of a and b.

Q12. If A is a square matrix such that $A^2=I$, then find value of $(A-I)^3+(A+I)^3-7A$.

Q13. If $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then find A^T-B^T.

Q14. Find a matrix A such that $2A-3B+5C=O$, where $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$.

SECTION – C (3 MARKS EACH)

Q15. Find matrix A such that

$$\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{bmatrix}$$

OR

Express the following matrix as the sum of a symmetric and skew symmetric matrix.

$$A=\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$$

Q16. If $A=\begin{bmatrix}-3 & 2\\ 1 & -1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find scalar k so that $A^2+I=kA$.

Q17. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then prove that $A^2-4A-5I=O$.

Q18. Find the value of $x+y$ from the following equation: $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$

ANSWERS

MATRICES

Q. NO.	ANSWER

Q1	c	**Q2**	a
Q3	d	**Q4**	c
Q5	b	**Q6**	a
Q7	d	**Q8**	d
Q9	c	**Q10**	d

11	$a = \frac{-2}{3}$ and $b = \frac{3}{2}$
12	A
13	$\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$
14	$A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$
15	$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ **OR** $\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$
16	$k = -4$
17	(to show)
18	11

DETERMINANTS

SECTION – A (1 MARK EACH)

Q1. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is :

(A) 0 (B) 10

(C) $\mathbb{R} - \{10\}$ (D) $\mathbb{R} - \{-10\}$

Q2. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

Q3. If A is a square matrix of order 3 such that the value of $|\text{adj}\cdot A| = 8$, then the value of $|A^T|$ is :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) 8 (D) $2\sqrt{2}$

Q4. If a_{ij} and A_{ij} represent the $(ij)^{th}$ element and its cofactor of $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ respectively, then the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ is :

(A) 0 (B) – 28

(C) 114 (D) – 114

Q5. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

(A) –4 (B) 1

(C) 3 (D) 4

	Given below are two statements: one is labelled as **Assertion A** and other is labelled as **Reason R.** In the light of the given statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
Q6.	*Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $\lvert A \rvert \in [2, 4]$. *Reason (R) :* $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

SECTION – B (2 MARKS EACH)

Q7.	Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.
Q8.	Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$.
Q9.	If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

SECTION – C (3 MARKS EACH)

Q10.	If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A' A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.
Q11.	If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$, find the value of $(a + x) - (b + y)$.

Q12.	If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}.

SECTION – D (5 MARKS EACH)

Q13.	Using the matrix method, solve the following system of linear equations : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$ **OR** Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

SECTION – E (4 MARKS – CASE STUDY)

Q14. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

ANSWERS

DETERMINANTS

Q. NO.	ANSWER
1	D
2	D
3	D
4	A
5	D
6	A
7	(to show)
8	$\frac{1}{2}$
9	$x = 2$
10	(to show)
11	3
12	$A^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$
13	$x = 2, y = 3, z = 5$ OR $x = 3, y = -2, z = -1$
14	$(i) \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 180 \\ 50 \end{bmatrix}$ (ii) consistent $(iii)(a)\ x = 20, y = 30$ (b) Rs.170000

CONTINUITY & DIFFERENTIABILTY

SECTION – A (1 MARK EACH)

Q1. The function $f(x) = |x| + |x-2|$ is

(a) continuous, but not differentiable at $x = 0$ and $x = 2$
(b) differentiable but not continuous at $x = 0$ and $x = 2$
(c) continuous but not differentiable at $x = 0$ only
(d) neither continuous nor differentiable at $x = 0$ and $x = 2$

Q2. Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is :

(A) $\mathbb{R}$ (B) 1

(C) 0 (D) –1

Q3. The derivative of 2^x w.r.t. 3^x is :

(A) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ (B) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$

(C) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ (D) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

Q4. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :

(A) –1 (B) 1

(C) –e (D) $-\frac{1}{e}$

Q5. If $y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is :

(A) $\frac{1}{\sqrt{e^{-2x}+1}}$ (B) $-\frac{1}{\sqrt{e^{-2x}+1}}$

(C) $\frac{1}{\sqrt{e^{-2x}-1}}$ (D) $-\frac{1}{\sqrt{e^{-2x}-1}}$

	Given below are two statements: one is labelled as **Assertion A** and other is labelled as **Reason R.** In the light of the given statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
Q6.	**Assertion(A):** f(x)=[x]is not differentiable at x=2. **Reason(R):** f(x)=[x] is not continuous at x=2.

SECTION – B (2 MARKS EACH)

Q7.	Find the values of a and b so that the following function is differentiable for all values of x : $f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \le -1 \end{cases}$.
Q8.	If $f(x) = \lvert \tan 2x \rvert$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.
Q9.	If $x = \sqrt{a^{\tan^{-1} t}}$, $y = \sqrt{a^{\cot^{-1} t}}$, then show that $x\frac{dy}{dx} + y = 0$.

SECTION – C (3 MARKS EACH)

Q10.	Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\frac{dy}{dx}$.
	OR If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.
Q11.	If $x^{30} y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.
Q12.	If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$.

SECTION – D (5 MARKS EACH)

Q13.

(a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

(b) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

SECTION – E (4 MARKS – CASE STUDY)

Q14

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Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

(iii) (a) Check if the function f(x) is differentiable at x = 1. *2*

OR

(iii) (b) Find f'(2) and f'(– 1). *2*

ANSWERS

CONTINUITY & DIFFERENTIABILTY

Q. NO.	ANSWER
1	A
2	D
3	C
4	A
5	D
6	A
7	$a=3, b=-\frac{3}{2}$
8	-8
10	$\frac{dy}{dx}=(\sin x)^x.x^{\sin x}\left[x\cot x+\log\sin x+\frac{\sin x}{x}+\log x\cos x\right]+a^x\log a$ **OR** $\frac{dy}{dx}=\cot 2t$
11	To prove
12	To show
13	$\frac{1}{2}$ **OR** to prove
14	$(i)-1 \quad (ii)-1$ $(iii)(a)\ f$ is diff. at $x=1 \quad (iii)(b)-1$ and -2

APPLICATIONS OF DERIVATIVES

SECTION – A (1 MARK EACH)

Q1. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(A) –60 units/sec (B) 60 units/sec

(C) –70 units/sec (D) –140 units/sec

Q2. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(A) 2 (B) 1

(C) 0 (D) –2

Q3. The point of inflexion of a function f(x) is the point where

(A) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point.

(B) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point.

(C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.

(D) $f'(x) \neq 0$.

Q4.	Given below are two statements: one is labelled as **Assertion A** and other is labelled as **Reason R.** *Assertion (A) :* The function $f(x) = x^2 - x + 1$ is strictly increasing on $(-1, 1)$. *Reason (R) :* If $f(x)$ is continuous on $[a, b]$ and derivable on (a, b), then $f(x)$ is strictly increasing on $[a, b]$ if $f'(x) > 0$ for all $x \in (a, b)$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct

SECTION – B (2 MARKS EACH)

Q5.	A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a.
Q6.	If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

SECTION – C (3 MARKS EACH)

Q7.	Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
Q8.	Find the intervals in which the following function f is strictly increasing or strictly decreasing : $f(x) = 20 - 9x + 6x^2 - x^3$.
Q9.	Find the absolute maximum and minimum values of the function $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [0, 1]$.

SECTION – D (5 MARKS EACH)

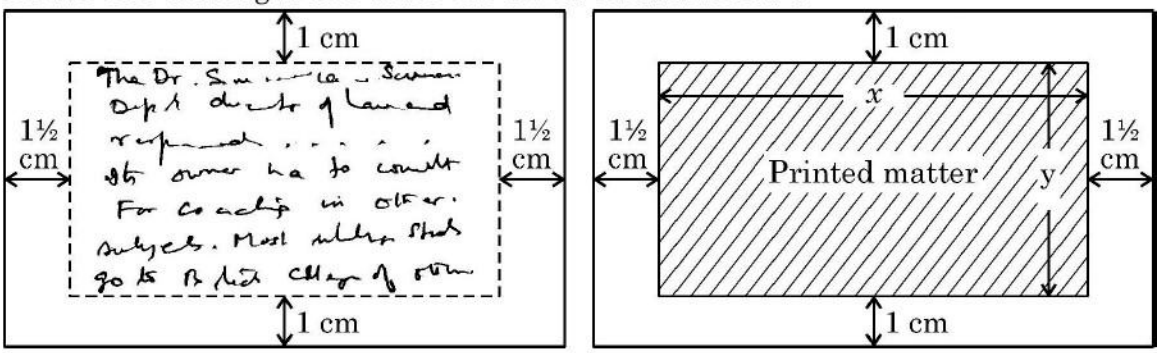
Q10. The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

OR

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

SECTION – E (4 MARKS – CASE STUDY)

Q11. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :

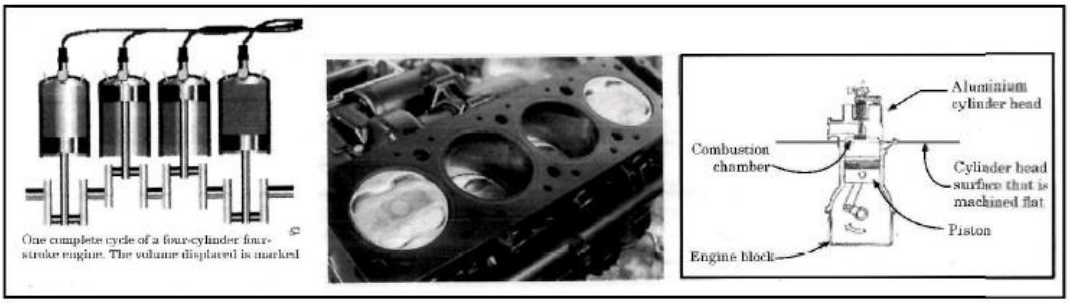


On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

Q12. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. **1**

(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h > r. State true or false and justify. **2**

SELF ASSESSMENT TEST 2024-25

ANSWERS

APPLICATIONS OF DERIVATIVES

Q. NO.	ANSWER
1	A
2	A
3	C
4	D
5	$a = 2$
6	–4
7	To show
8	f(x) is strictly increasing in (1,3) or [1,3] And f(x) is strictly decreasing in $(-\infty, 1) \cup (3, \infty)$ or $(-\infty, 1] \cup [3, \infty)$
9	absolute maximum value is 6 and absolute minimum value is $-\frac{9}{4}$
10	Width = 100 cm , height = 50 cm
11	$(i)\ A = 30 + 2x + \frac{72}{x}$ (ii) Length = 9 cm, breadth = 6 cm
12	$(i)\ V = \frac{\pi}{2}\left(75r - r^3\right)$ $(ii)\ \frac{dV}{dr} = \frac{\pi}{2}\left(75 - 3r^2\right)$ $(iii)(a)\ r = 5\text{cm}$ (b) False