St. XAVIER'S HIGH SCHOOL

MATHEMATICS (041)

WORKSHEET 6 OF CHAPTER 6(2026-27)

GRADE- XII

1 The interval in which the function f defined by $f(x) = e^x$ is strictly increasing ,is

(A) $[1,\infty)$ $(B)(-\infty, 0)$ $(B)(-\infty,\infty)$ $(B)(0,\infty)$

$[Ans - c]$

2 The interval on which the function $f(x) = 2x^3 + 9x^2 + 12\text{x} - 1$ is decreasing is

(A) only at $x = 0$ (B) only at $x = 1$ (C) in R (D) none of these

$[Ans - c]$

3 The radius of a cylinder is increasing at the rate of 3 cm/sec and its height is decreasing at the rate of 4 cm/sec. The rate of change of its volume when radius is 4 cm and height is 6 cm, is

(A) $64cm^3/sec$ (B) $144cm^3/sec$ (C) $80m^3/sec$ (D) $80\pi cm^3/sec$

$[Ans - D]$

4 The function $f(x) = \tan x - x$

(A) Always increases (B) always decreases

(C) remains constant (D) sometimes increases and sometimes decreases

$[Ans - D]$

5 At $x = \frac{5\pi}{6}$, the function $f(x) = 2\sin 3x + 3\cos 3x$ is

(A) Maximum (B) minimum (C) zero (D) neither maximum nor minimum

$[Ans - A]$

6 The function $y = x^2e^{-x}$ is increasing in the interval

(A) $[0,2]$ (B) $[2,\infty)$ (C) $(-\infty, 0]$ (D)$(-\infty, 0] \cup [2, \infty)$

$[Ans - D]$

7 If $f(x) = a(x - cosx)$ is strictly decreasing in 'R', then a belongs to

(A) $\{0\}$ (B) $(0,\infty)$ (C) $(-\infty, 0)$ (D)$(-\infty, \infty)$

$[Ans - C]$

8 The function $f(x) = kx - \sin x$ is strictly increasing for

(A) k> 1 (B) k< 1 (C) k> -1 (D)k< -1

$[Ans - A]$

10 If $f(x) = ax^2 + 6x + 5$ attains the maximum value at $x = 1,$ then the value of a is

(A) 0 (B) 5 (C) 3 (D) -3

11 If the area of a circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

12 The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when two equal sides are equal to the base?

13 The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of an edge of the cube.

14 A man 2m tall, walks at the rate of $1\frac{2}{3}$ m/sec towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is $3\frac{1}{3}$ m from the base of the light?

15 A kite is moving horizontally at a height of 151.5 metres. If the speed of the kite is 10m/sec, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite? The height of the boy is 1.5 m.

16 Two men A and B start with velocities v at the same time from junction of two roads inclined at 45^0 to each other. If they travel by different roads, find the rate at which they are being separated.

17 A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

$[Ans - 160cm/sec]$

18 Find the intervals in which the following functions are strictly increasing or strictly decreasing

i. $f(x) = 10 - 6x - 2x^2$ ii. $f(x) = 20 - 9x + 6x^2 - x^3$

$$\begin{bmatrix} Ans - i. \left[\frac{-3}{2}, \infty\right) \\ ii. (-\infty, 1] \cup [3, \infty) \end{bmatrix}$$

19 Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$

$[Ans - (2,29) and\ (3,28)]$

20 Find the sub-intervals in which $f(x) = \log(2+x) - \frac{x}{2+x}, x > -2$ is increasing or decreasing.

21 Show that the function f defined by $f(x) = tan^{-1}(\sin x + \cos x)$ is strictly increasing in the interval$\left(0, \frac{\pi}{4}\right)$.

22 Find the intervals in which the function $f(x) = \frac{x}{\log x}$ is increasing or decreasing.

23 Find all the points of local maxima and minima an d the corresponding maximum and minimum values of the function $f(x) = -\frac{3}{4}x^4 - 8x^3 \text{-} \frac{45}{2}x^2 + 105$.

24 Find all the points of local maxima and minima an d the corresponding maximum and minimum values of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$

25 If M and m denote local maximum and minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$respectively, find the value of (M-m)

26 Find the local maximum and local minimum values of $f(x) = \sec x + logcos^2 x, 0 < x < 2\pi$

27 Prove that $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $x = \frac{\pi}{6}$

28 Find the local maximum and local minimum values (which ever exists) for the function

$f(x) = 4x^2 + \frac{1}{x}, x \neq 0.$

29 Find both the maximum and minimum values of $3x^4 - 8x^3 + 12x^2 - 48x + 1$ on the interval $[1,4]$.

30 Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval$[1,2]$.

31 Find the maximum and minimum values of $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \frac{\pi}{2}\right]$

32 Find the minimum values of $ax + by$,where $xy = c^2$ anda, b, c are positive.

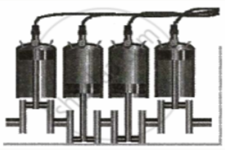
33 A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum?

34 A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

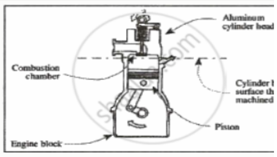
35 If $f(x) = \frac{1}{4x^2+2x+1}$; $x \in R$,then find the maximum value of $f(x)$.

36 Find the least value of the function $f(x) = ax + \frac{b}{x}, (a > 0, b > 0, x > 0)$.

37 Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.



One complete of a four-cylinder four-stroke engine. The volume displace is marked



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions:

i. If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. (1)

ii. Find $\frac{dV}{dr}$. (1)

iii. (a) Find the radius of cylinder when its volume is maximum. (2)

OR

(b) For maximum volume, $h > r$. State true or false and justify. (2)