GRADE 9 MATHEMATICS

Chapter 5: I’m Up and Down, and Round and Round

Exercises 5.2–5.5 + Theorems 4–12

Easy, step-by-step solutions for notebook writing

Learning Sequence

Follow the same order as the Ganita Manjari chapter.

- Exercise 5.2 — chord and centre
- Theorem 4 → Exercise 5.3
- Theorem 5 → Exercise 5.4
- Theorem 6 → Exercise 5.4
- Theorem 7 → Theorem 8
- Exercise 5.5 — chord length

- Theorem 9 — angle subtended by an arc
- Angle in a semicircle
- Definition: concyclic points
- Theorem 10 — concyclicity
- Theorem 11 — cyclic quadrilateral
- Theorem 12 — converse

Exercise 5.2 — Question 1

Show that the triangle formed by a chord and the centre of the circle is isosceles.

Given: AB is a chord of a circle with centre O. Join OA and OB.

OA = OB (radii of the same circle)

Therefore, two sides of ΔOAB are equal.

By definition, a triangle with two equal sides is isosceles.

∴ ΔOAB is an isosceles triangle. Hence proved.

Exercise 5.2 — Question 2

Show that if two such isosceles triangles have equal base length, they are congruent.

Given: AB and CD are equal chords of the same circle with centre O.

OA = OC (radii of the same circle)

OB = OD (radii of the same circle)

AB = CD (given)

Therefore, ΔOAB ≅ ΔOCD by SSS congruence.

∴ The two isosceles triangles are congruent. Hence proved.

Theorem 4

The line joining the centre of a circle and the midpoint of a chord is perpendicular to the chord.

Given
AB is a chord; M is its midpoint; O is the centre.

To prove
OM ⊥ AB

Solution / Proof (write in this order)

Join OA and OB.

AM = BM (M is the midpoint of AB).

OA = OB (radii of the same circle).

OM = OM (common side).

Therefore, ΔOMA ≅ ΔOMB by SSS congruence.

Thus ∠OMA = ∠OMB (CPCT).

These two angles form a straight angle, so each is 90°.

∴ OM ⊥ AB. Hence proved.

Exercise 5.3 — Question 1

Why does the perpendicular from the centre of a circle to a chord bisect the chord?

Given: AB is a chord, O is the centre and $OM \perp AB$.

Join OA and OB.

$OA = OB$ (radii).

$OM = OM$ (common side).

$\angle OMA = \angle OMB = 90°$.

Therefore, $\Delta OMA \cong \Delta OMB$ by RHS congruence.

By CPCT, $AM = BM$.

∴ OM bisects AB. Hence proved.

Exercise 5.3 — Question 2

An isosceles triangle ABC is inscribed in a circle, with AB = AC. Show that the altitude from A to BC passes through the centre.

Let the altitude from A meet BC at M.

Since AB = AC, ΔABC is isosceles.

Therefore, the altitude AM also bisects BC.

So BM = CM; hence M is the midpoint of chord BC.

By Theorem 4, the line joining the centre O to M is perpendicular to BC.

AM is also perpendicular to BC at M.

Through a point, only one perpendicular can be drawn to a line.

Therefore O lies on AM.

∴ The altitude from A to BC passes through the centre O.

Exercise 5.3 — Question 3

Two parallel chords of lengths 6 cm and 8 cm are on opposite sides of the centre. Radius = 5 cm. Find the distance between their midpoints.

.

$\therefore$

Theorem 5

The perpendicular from the centre of a circle to a chord bisects the chord.

Given
AB is a chord of a circle with centre O; $OM \perp AB$.

To prove
$AM = BM$

Solution / Proof (write in this order)

Join OA and OB.

$OA = OB$ (radii of the same circle).

$OM = OM$ (common side).

$\angle OMA = \angle OMB = 90°$.

Therefore, $\Delta OMA \cong \Delta OMB$ by RHS congruence.

By CPCT, $AM = BM$.

∴ The perpendicular from the centre bisects the chord. Hence proved.

Exercise 5.4 — Question 1

Use the Baudhāyana–Pythagoras theorem to show why Theorem 6 is true.

Let AB and CD be equal chords of a circle with centre O.

Drop $OM \perp AB$ and $ON \perp CD$.

By Theorem 5, $AM = AB/2$ and $CN = CD/2$.

Since $AB = CD$, $AM = CN$.

Also $OA = OC = r$.

In right ΔOMA: $OA^2 = OM^2 + AM^2$.

In right ΔONC: $OC^2 = ON^2 + CN^2$.

Since $OA = OC$ and $AM = CN$, we get $OM^2 = ON^2$.

Therefore $OM = ON$.

∴ Equal chords are equidistant from the centre.

Theorem 6

Chords of equal length are equidistant from the centre.

Given
AB = CD are two chords of the same circle.

To prove
Distance from O to AB = distance from O to CD

Solution / Proof (write in this order)

Let OM $\perp$ AB and ON $\perp$ CD.

By Theorem 5, M and N are the midpoints of the chords.

Therefore AM = AB/2 and CN = CD/2.

Since AB = CD, AM = CN.

OA = OC (radii).

By Pythagoras in the two right triangles, $OM^2 = ON^2$.

Hence OM = ON.

∴ Equal chords are equidistant from the centre. Hence proved.

Exercise 5.4 — Question 2

In Fig. 5.15, CE ⊥ AB, CH ⊥ GF and CE = CH. Show that AB = GF.

CE ⊥ AB and CH ⊥ GF.

By Theorem 5, E and H are midpoints of the chords.

Therefore AE = EB and GH = HF.

CA = CG (radii of the same circle).

CE = CH (given).

∠CEA = ∠CHG = 90°.

Therefore ΔCEA ≅ ΔCHG by RHS congruence.

So AE = GH (CPCT).

Thus AB = 2AE and GF = 2GH.

∴ AB = GF. Hence proved.

Exercise 5.4 — Question 3

Solve the previous question using the Baudhāyana–Pythagoras theorem.

In right ΔCEA: $CA^2 = CE^2 + AE^2$.

In right ΔCHG: $CG^2 = CH^2 + GH^2$.

$CA = CG = r$ and $CE = CH$.

Therefore $r^2 = CE^2 + AE^2 = CH^2 + GH^2$.

So $AE^2 = GH^2$.

Hence $AE = GH$.

Since $AB = 2AE$ and $GF = 2GH$, we get $AB = GF$.

$\therefore$ $AB = GF$. Hence proved by Pythagoras.

Theorem 7

Chords of a circle that are equidistant from the centre have equal length.

Given

$CE \perp AB$, $CH \perp GF$ and $CE = CH$.

To prove

$AB = GF$

Solution / Proof (write in this order)

By Theorem 5, E and H are midpoints of AB and GF.

$CA = CG$ (radii).

$CE = CH$ (given).

$\angle CEA = \angle CHG = 90°$.

Thus $\Delta CEA \cong \Delta CHG$ by RHS.

So $AE = GH$.

$AB = 2AE$ and $GF = 2GH$.

$\therefore AB = GF$. Hence proved.

Theorem 8

Of two unequal chords, the longer chord is closer to the centre.

Given
AB and DE are chords with $AB > DE$.

To prove
Distance CF < distance CG, where $CF \perp AB$ and $CG \perp DE$.

Solution / Proof (write in this order)

By Theorem 5, F and G are midpoints.

Therefore $AF = AB/2$ and $DG = DE/2$.

Since $AB > DE$, $AF > DG$.

$OA = OD = r$.

By Pythagoras: $OA^2 = CF^2 + AF^2$ and $OD^2 = CG^2 + DG^2$.

So $CF^2 + AF^2 = CG^2 + DG^2$.

Since $AF^2 > DG^2$, we get $CF^2 < CG^2$.

Therefore $CF < CG$.

∴ The longer chord is nearer to the centre. Hence proved.

Exercise 5.5 — Question 1

Find the length of a chord when radius = 7 cm and perpendicular distance from the centre = 6 cm.

Let AB be the chord and $OM \perp AB$.

By Theorem 5, M is the midpoint of AB.

OA = 7 cm and OM = 6 cm.

In right ΔOMA, $OA^2 = OM^2 + AM^2$.

$7^2 = 6^2 + AM^2$.

$AM^2 = 49 - 36 = 13$.

$AM = \sqrt{13}$ cm.

$AB = 2AM = 2\sqrt{13}$ cm.

∴ Chord length = $2\sqrt{13}$ cm ≈ 7.21 cm.

Exercise 5.5 — Question 2

Explain why chord length = $2\sqrt{(r^2 - d^2)}$, where r is radius and d is perpendicular distance from the centre.

Let AB be a chord and $OM \perp AB$.

By Theorem 5, M is the midpoint of AB, so AB = 2AM.

OA = r and OM = d.

In right ΔOMA, by Pythagoras:

$OA^2 = OM^2 + AM^2$

$r^2 = d^2 + AM^2$

$AM^2 = r^2 - d^2$

$AM = \sqrt{(r^2 - d^2)}$.

Therefore $AB = 2AM = 2\sqrt{(r^2 - d^2)}$.

∴ Chord length = $2\sqrt{(r^2 - d^2)}$.

Exercise 5.5 — Question 3*

If the distance of chord AB from the centre is twice the distance of chord CD, can we conclude CD = 2AB?

Let the distances be d_1 and d_2, with $d_1 = 2d_2$.

Chord length is $2\sqrt{(r^2 - d^2)}$.

So $AB = 2\sqrt{(r^2 - d_1^2)}$ and $CD = 2\sqrt{(r^2 - d_2^2)}$.

The relation between distances is inside a square root.

Therefore doubling the distance does NOT double the chord length.

Example: let $r = 5$, $d_2 = 2$, $d_1 = 4$.

Then $CD = 2\sqrt{21}$ and $AB = 2\sqrt{9} = 6$ (depending on which chord has which distance).

They are not in the ratio 2:1.

∴ No. We cannot conclude CD = 2AB. The statement is false in general.

(*) Starred question from the textbook.

Theorem 9

The angle subtended by an arc at the centre is double the angle subtended by the same arc at any point on the remaining circle.

Given
Arc AB subtends ∠AOB at centre O and ∠ACB at point C on the remaining circle.

To prove
∠AOB = 2∠ACB

Solution / Proof (write in this order)

Join OA, OB and OC.

OA = OC and OB = OC because all are radii.

Therefore the relevant triangles are isosceles.

Let ∠ACO = x and ∠OCB = y.

Then the central angle ∠AOB is $2x + 2y$.

The angle at C is $x + y$.

Hence ∠AOB = $2(x + y)$ = 2∠ACB.

∴ Central angle = 2 × angle at the circle.

Corollary — Angle in a Semicircle

Show that the angle subtended by a diameter at any point on the circle is 90°.

Let AB be a diameter and C be any point on the circle.

Since AB is a diameter, the central angle $\angle AOB = 180°$.

By Theorem 9, $\angle ACB = \frac{1}{2}\angle AOB$.

Therefore $\angle ACB = \frac{1}{2} \times 180° = 90°$.

∴ The angle in a semicircle is always 90°.

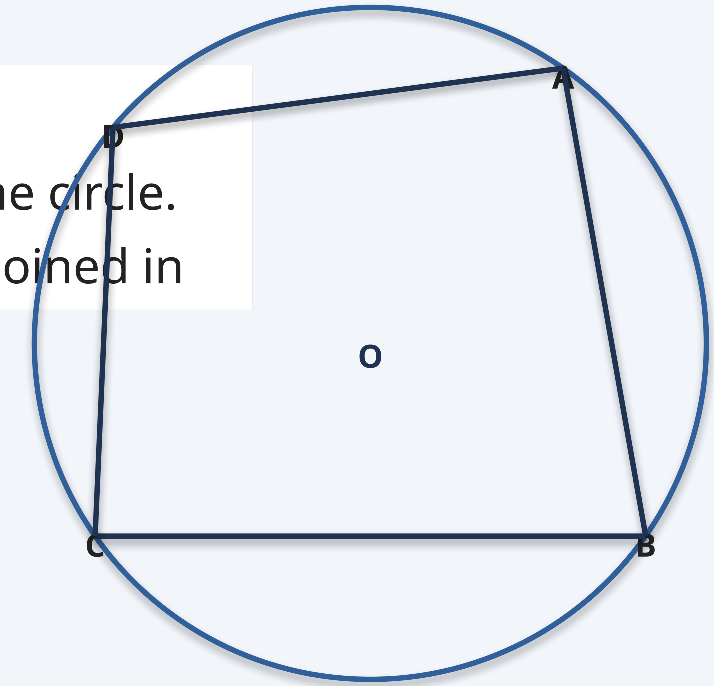
Definition — Concyclicity of Points

Points are called CONCYCLIC if all of them lie on the same circle.

Any 3 non-collinear points determine a unique circle.

A fourth point is concyclic with them if it lies on that same circle.

Four concyclic points form a **cyclic quadrilateral** when joined in order.



A, B, C, D are concyclic.

Theorem 10

If a line segment AB subtends equal angles at two points C and D on the same side of AB, then A, B, C, D are concyclic.

Given
$\angle ACB = \angle ADB$; C and D lie on the same side of AB.

To prove
All four points lie on one circle.

Solution / Proof (write in this order)

Draw the circle through A, B and C.

Suppose D is not on this circle.

Let AD meet the circle again at E.

Since C and E lie on the same segment of chord AB, $\angle ACB = \angle AEB$.

Given $\angle ACB = \angle ADB$.

So $\angle AEB = \angle ADB$.

But $\angle AEB$ is an exterior angle of ΔBED, so $\angle AEB > \angle ADB$.

This is a contradiction.

Therefore D must lie on the circle.

$\therefore$ A, B, C, D are concyclic. Hence proved.

Theorem 11

The sum of two opposite angles of a cyclic quadrilateral is 180°.

Given
ABCD is a cyclic quadrilateral.

To prove
$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

Solution / Proof (write in this order)

Let O be the centre of the circle.

By Theorem 9, $\angle A = \frac{1}{2}$(central angle subtending arc BCD).

Similarly, $\angle C = \frac{1}{2}$(central angle subtending arc BAD).

The two arcs together make the full circle, so their central angles total 360°.

Therefore $\angle A + \angle C = \frac{1}{2} \times 360° = 180°$.

Similarly, $\angle B + \angle D = 180°$.

∴ Opposite angles of a cyclic quadrilateral are supplementary.

Theorem 12

If two opposite angles of a quadrilateral add up to 180°, then its vertices are concyclic.

Given
ABCD is a quadrilateral and ∠A + ∠C = 180°.

To prove
ABCD is cyclic.

Solution / Proof (write in this order)

Draw the circle through A, B and D.

Let CD meet this circle again at E.

ABED is cyclic, so by Theorem 11: ∠A + ∠BED = 180°.

Given: ∠A + ∠C = 180°.

Therefore ∠BED = ∠C.

But ∠BED is an exterior angle of ΔBCE, so it is greater than ∠C.

This is impossible.

Therefore C must lie on the circle through A, B and D.

∴ A, B, C, D are concyclic. Hence proved.

Theorem 11 — Quick Application

In a cyclic quadrilateral, $\angle A = 110°$. Find the opposite angle $\angle C$.

By Theorem 11, opposite angles of a cyclic quadrilateral are supplementary.

Therefore $\angle A + \angle C = 180°$.

$110° + \angle C = 180°$.

$\angle C = 180° - 110° = 70°$.

$\therefore \angle C = 70°$.

Theorem 12 — Quick Application

A quadrilateral has opposite angles 105° and 75°. Can its four vertices lie on a circle?

Check the sum of opposite angles.

$105° + 75° = 180°$.

By Theorem 12, if opposite angles add to 180°, the four vertices are concyclic.

∴ Yes. The quadrilateral is cyclic.

One-Page Revision — Theorems 4 to 12

Theorem 4: Centre → midpoint of chord is perpendicular to chord.

Theorem 5: Perpendicular from centre → bisects chord.

Theorem 6: Equal chords → equal distances from centre.

Theorem 7: Equal distances from centre → equal chords.

Theorem 8: Longer chord → nearer to centre.

Theorem 9: Central angle = 2 × angle at circumference.

Theorem 10: Equal angles on same side of a segment → concyclic points.

Theorem 11: Opposite angles of cyclic quadrilateral = 180°.

Theorem 12: Opposite angles sum to 180° → quadrilateral is cyclic.

Exam tip: Always write Given → To Prove → Proof → Therefore/Hence Proved.

WRITE • UNDERSTAND • PRACTISE

Use the proof steps as a model,
then practise drawing the required figure neatly.